LECTURE L2-11

Nuclear Chemistry

Half-Life and Binding Energy: Why Bananas Are Radioactive and How Smoke Detectors Work

Duration: 75 minutes

Advanced lecture script — pre-requisite: Level 1

HOOK (3 minutes)

Teacher holds up (or shows photos of):

A smoke detector (contains americium-241)

A banana (contains potassium-40)

A radon test kit ($15-30 at hardware store)

A glow-in-the-dark watch (pre-1970s — radium)

Teacher says: "A banana is radioactive. A smoke detector is radioactive. Your granite countertop is radioactive. But you're not glowing green or getting sick.

How much radiation is actually dangerous? And how do scientists calculate half-life to date ancient artifacts?

- Today's question: How do you calculate radioactive decay — and what levels are safe? *

By the end of this session, you will be able to:

Calculate half-life and remaining radioactive material

Understand the difference between alpha, beta, and gamma radiation quantitatively

Use the decay constant (λ) and decay equation ($N = N_0e^{-tt}$)

Calculate binding energy and understand why some atoms are radioactive

Distinguish between fission and fusion (nuclear power vs. the sun)"

SEGMENT 1: Review from Level 1 and Previous Level 2 Lectures (5 minutes)

Teacher says: "Before we go deeper, let's recall what you already know."

Review from Level 1 (Nuclear Chemistry - Basic)

Level 1 Concept	Definition	Household Example
Alpha (α)	Heavy, slow particle — stopped by paper	Smoke detector (safe when sealed)
Beta (β)	Fast electron — stopped by aluminum foil	Carbon-14 dating
Gamma (γ)	High-energy wave — stopped by lead/concrete	Medical X-rays
Half-life	Time for half of radioactive atoms to decay	Carbon-14: 5,730 years

Level 1 Concept	Definition	Household Example
Radon	Radioactive gas from soil — alpha emitter	Test your home ($15-30 kit)
Trefoil symbol (☢)	Warning: radioactive material present	On smoke detectors, medical devices

Review from Level 2-9 (Kinetics - First Order)

Level 2-9 Concept	Formula	Connection to Nuclear
First-order half-life	$t_{1/2} = 0.693/k$	Radioactive decay is first-order
Rate constant (k)	$k = 0.693 \div t_{1/2}$	Used in decay calculations

> Quick check (show of hands / chat): "What type of radiation is stopped by paper?" (Alpha) *"What is the half-life of carbon-14?"* (5,730 years) "Should you test your home for radon?" (Yes — test kits are cheap)

Teacher: "Good. Now let's learn how to calculate decay and understand why some atoms are radioactive."

SEGMENT 2: Nuclear Decay Equations — Alpha, Beta, Gamma (10 minutes)

> Teacher says: "When a radioactive nucleus decays, it changes into a different element. We write nuclear equations to show this."

Alpha Decay

Alpha decay: Nucleus emits an alpha particle ($^{4}_{2}He$) — 2 protons + 2 neutrons. Atomic number decreases by 2, mass number decreases by 4.

General form: ^A_Z X → ^(A-4)_(Z-2) Y + ^4_2 He

Worked Example: Americium-241 (Smoke Detector)

Step	Equation
Americium-241 decays to Neptunium-237	^241_95 Am → ^237_93 Np + ^4_2 He

Teacher: *"Americium-241 has a half-life of 432 years. That's why smoke detectors last so long — they remain radioactive for decades."*

Beta Decay

Beta decay: A neutron turns into a proton and emits an electron (beta particle). Atomic number increases by 1, mass number stays the same.

General form: $^{A}_{Z}X \rightarrow ^{A}_{(Z+1)}Y + ^{0}_{-1}e$ (or β^-)

Worked Example: Carbon-14 (Radiocarbon Dating)

Step	Equation
Carbon-14 decays to Nitrogen-14	$^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e$

Teacher: *"Carbon-14 has a half-life of 5,730 years. By measuring how much C-14 remains in ancient bones or wood, scientists can date them up to about 50,000 years old."*

Gamma Decay

Gamma decay: Nucleus releases energy (gamma photon) without changing element. Atomic number and mass number stay the same.

General form: $^{A}_{Z}X^* \rightarrow ^{A}_{Z}X + \gamma$

Teacher: "Gamma rays are the most penetrating type of radiation. They are used in medical imaging (X-rays, CT scans) and cancer treatment (radiation therapy)."

Partner talk (1 minute): *"Tell your partner: Uranium-238 ($^{238}_{92}U$) undergoes alpha decay. What is the new element? (Thorium-234 — $^{234}_{90}Th$)."*

SEGMENT 3: Half-Life Calculations — How Much Remains? (12 minutes)

Teacher says: "Half-life is constant for a given isotope. The formula for remaining amount after n half-lives is:"

Remaining = Initial × $(1/2)^n$

Where n = number of half-lives = time ÷ half-life

Worked Example 1: Carbon-14 Dating (Simple)

Problem: A bone initially had 100 g of carbon-14. The half-life of C-14 is 5,730 years. How much C-14 remains after 17,190 years?

Step	Calculation
Number of half-lives (n) = 17,190 ÷ 5,730	= 3 half-lives
Remaining = 100 g × $(1/2)^3$	= 100 g × 1/8 = 12.5 g

Answer: 12.5 grams remain.

Worked Example 2: Finding Age from Remaining Amount

Problem: A wooden artifact has 25% of its original carbon-14 remaining. How old is it? ($t_{1/2}$ = 5,730 years)

Step	Calculation
25% = 1/4 = $(1/2)^2$	So n = 2 half-lives
Age = 2 × 5,730 years	= 11,460 years

Answer: The artifact is about 11,460 years old.

Worked Example 3: Potassium-40 in Bananas

Problem: A banana contains about 0.5 g of potassium. About 0.0117% of that is potassium-40 (radioactive). Half-life of K-40 = 1.25×10^9 years (1.25 billion years). How much K-40 remains after 2.5 billion years?

Step	Calculation
n = $2.5 \times 10^9 \div 1.25 \times 10^9$	= 2 half-lives
Remaining fraction = $(1/2)^2$	= 1/4 = 25%
Initial K-40 mass = 0.5 g × 0.000117	= 5.85×10^{-5} g
Remaining = 5.85×10^{-5} g × 0.25	= 1.46×10^{-5} g

Answer: About 1.46×10^{-5} g of K-40 remains — still detectable by a Geiger counter.

Worked Example 4: Radon-222 (Home Safety)

Problem: Radon-222 has a half-life of 3.8 days. If a basement has 100 pCi/L (picocuries per liter) of radon today, how much remains after 7.6 days?

Step	Calculation
n = 7.6 ÷ 3.8	= 2 half-lives
Remaining = 100 × $(1/2)^2$	= 100 × 1/4 = 25 pCi/L

Answer: 25 pCi/L. But note: Radon is constantly being produced from uranium in soil, so levels may not drop that much in a real basement without mitigation.

Partner talk (1 minute): *"Tell your partner: Iodine-131 (used in medical imaging) has a half-life of 8 days. After 24 days, what fraction remains? (24 ÷ 8 = 3 half-lives → $(1/2)^3$ = 1/8 remains)."*

SEGMENT 4: The Decay Constant (λ) and Exponential Decay Formula (10 minutes)

Teacher says: "For more precise calculations, chemists use the exponential decay formula:"

$N = N_0 \times e^{-tt}$

Where:

N = amount remaining

N_0 = initial amount

λ (lambda) = decay constant

t = time

e = 2.718... (base of natural logarithm)

Relationship between half-life and decay constant:

$t_{1/2} = \ln(2) \div \lambda = 0.693 \div \lambda$ $\lambda = 0.693 \div t_{1/2}$

Worked Example 1: Finding λ from Half-Life

Problem: Carbon-14 has $t_{1/2}$ = 5,730 years. What is λ?

Step	Calculation
$\lambda = 0.693 \div t_{1/2}$	= 0.693 ÷ 5,730 = 1.21×10^{-4} yr^{-1}

Answer: λ = 1.21×10^{-4} yr^{-1}.

Worked Example 2: Using $N = N_0e^{-tt}$

Problem: A sample has 1,000 atoms of carbon-14 initially. How many remain after 10,000 years? (λ = 1.21×10^{-4} yr^{-1})

Step	Calculation
N = 1,000 × e^(-1.21 × 10^{-4} × 10,000)	
λt = 1.21 × 10^{-4} × 10,000	= 1.21
$e^{-1.21}$	= 0.298
N = 1,000 × 0.298	= 298 atoms

Answer: About 298 atoms remain.

Compare to half-life method: 10,000 ÷ 5,730 ≈ 1.745 half-lives. (1/2)^1.745 = 0.298 — same result.

Worked Example 3: Finding Time Using Natural Log

Problem: How long will it take for 90% of a radioactive sample to decay? ($t_{1/2}$ = 10 years)

Step	Calculation
If 90% decayed, then 10% remains: N/N_0 = 0.10	
$N/N_0 = e^{-tt} = 0.10$	

Step	Calculation
$\lambda = 0.693 \div 10 = 0.0693\ yr^{-1}$	
$e^{-0.0693t} = 0.10$	
Take natural log of both sides: $-0.0693t = \ln(0.10)$	
$\ln(0.10) = -2.3026$	
$-0.0693t = -2.3026$	
$t = 2.3026 \div 0.0693$	= 33.2 years

Answer: 33.2 years (about 3.3 half-lives).

Partner talk (1 minute): *"Tell your partner: If $\lambda = 0.01\ yr^{-1}$, what is the half-life? ($t_{1/2} = 0.693 \div 0.01 = 69.3$ years)."*

SEGMENT 5: Binding Energy — Why Some Atoms Are Radioactive (12 minutes)

Teacher says: "Binding energy is the energy that holds the nucleus together. It's the energy released when protons and neutrons come together to form a nucleus."

Binding energy = (mass defect) × c^2

Where c = speed of light (3.00×10^8 m/s)

Mass defect: The difference between the mass of the nucleus and the sum of its individual protons and neutrons. Some mass is converted to energy ($E = mc^2$).

The Most Stable Nucleus: Iron-56

Teacher: *"Iron-56 has the highest binding energy per nucleon (8.8 MeV per proton/neutron). That means it's the most stable nucleus."*

Element	Binding Energy per Nucleon (MeV)	Stability
Hydrogen-1	0 (no binding — single proton)	Unstable alone
Helium-4	7.1	Very stable
Carbon-12	7.7	Stable
Iron-56	8.8 (highest)	Most stable
Uranium-238	7.6	Radioactive

Teacher: "Elements lighter than iron can release energy by fusion (combining). Elements heavier than iron can release energy by fission (splitting). That's why the sun fuses hydrogen into helium, and nuclear power plants split uranium."

Binding Energy Calculation (Conceptual — No Math Required in Detail)

Teacher: "The famous equation $E = mc^2$ means mass and energy are interchangeable. When protons and neutrons come together to form a nucleus, some mass disappears — converted into binding energy."

Household example: *"The energy released in nuclear reactions is millions of times greater than chemical reactions. 1 gram of uranium-235 (fission) releases about 2-3 million times more energy than 1 gram of coal (combustion). That's why nuclear power plants use so little fuel."*

Fission vs. Fusion

Process	What Happens	Example	Household Connection
Fission	Heavy nucleus splits into lighter nuclei	^235_92 U + n → ^141_56 Ba + ^92_36 Kr + 3n + energy	Nuclear power plants, atomic bombs
Fusion	Light nuclei combine into heavier nucleus	^2_1 H + ^3_1 H → ^4_2 He + n + energy	Sun, hydrogen bombs

Teacher: "Fusion is what powers the sun — and if we can figure out how to control it on Earth, it could provide nearly limitless clean energy."

Partner talk (1 minute): *"Tell your partner: Which is more stable — iron-56 or uranium-238? (Iron-56 — it has the highest binding energy per nucleon)."*

SEGMENT 6: Radiation Doses — What's Safe? (8 minutes)

Teacher says: *"Radiation dose is measured in sieverts (Sv) . Background radiation is about 2-3 mSv per year."*

Typical Radiation Doses (Household Comparison)

Source	Dose (mSv)	Notes
Banana	0.0001	Extremely small — eat normally
Smoke detector (americium-241)	0.00001 per year	Safe — don't take apart

Source	Dose (mSv)	Notes
Chest X-ray	0.1	Low risk
CT scan (head)	2	About 1 year of background
Flight from NY to LA	0.04	More radiation at altitude
Annual background (natural)	2-3	From soil, radon, cosmic rays
Radon in home (4 pCi/L)	~20 per year	Action level — mitigate
Chernobyl nearby (1986)	100-500	Evacuation level
Acute radiation sickness	>1,000 (1 Sv)	Nausea, hair loss
LD50 (lethal dose 50%)	~4,000 (4 Sv)	Half of people die without treatment

Safety Rules Summary

Rule	Why
Test your home for radon	Radon is #1 cause of lung cancer in non-smokers
Don't take apart smoke detectors	Americium-241 is safe when sealed
Limit medical CT scans	Only when necessary
Don't wear old radium watches daily	Gamma exposure adds up over time
Distance is your friend	Radiation intensity drops by $1/\text{distance}^2$

Worked Example: Radiation Dose from Frequent Flights

Problem: A flight from NY to LA gives 0.04 mSv. How many flights would equal one chest X-ray (0.1 mSv)?

Step	Calculation
Number of flights = 0.1 mSv ÷ 0.04 mSv per flight	= 2.5 flights

Answer: About 2-3 cross-country flights equal one chest X-ray.

> Partner talk (1 minute): *"Tell your partner: What is the EPA action level for radon in homes? (4 pCi/L — above that, you should install a mitigation system)."*

SEGMENT 7: Putting It All Together — Complete Nuclear Problem (6 minutes)

Teacher says: "Let's do one complete problem that uses everything from today's lecture."

Problem: A sample of iodine-131 (used in medical imaging) has a half-life of 8.0 days. Initially, it has an activity (decay rate) of 100 mCi (millicuries).

What is the decay constant λ (in $days^{-1}$)?

After 24 days, what fraction of iodine-131 remains?

What is the activity after 24 days (in mCi)?

How long will it take for the activity to drop to 12.5 mCi?

Part 1: Decay constant λ

Step	Calculation
$\lambda = 0.693 \div t_{1/2}$	$= 0.693 \div 8.0 = 0.0866\ day^{-1}$

Answer Part 1: $\lambda = 0.0866\ day^{-1}$.

Part 2: Fraction remaining after 24 days

Step	Calculation
n = 24 ÷ 8 = 3 half-lives	
Fraction = $(1/2)^3$	= 1/8 = 0.125

Answer Part 2: 12.5% (1/8) remains.

Part 3: Activity after 24 days

Step	Calculation
Activity = 100 mCi × 1/8	= 12.5 mCi

Answer Part 3: 12.5 mCi.

Part 4: Time to drop to 12.5 mCi

Step	Calculation
12.5 mCi / 100 mCi = 0.125 = 1/8	
$(1/2)^n = 1/8 \rightarrow n = 3$ half-lives	
t = 3 × 8 days	= 24 days

Answer Part 4: 24 days.

CLOSING — The 60-Second Challenge (5 minutes)

Teacher says: *"Pair up. Person A: 60 seconds — explain the difference between alpha, beta, and gamma radiation (include what stops each). Person B: 60 seconds — a sample of strontium-90 ($t_{1/2}$ = 29 years) starts with 100 g. How much remains after 87 years? (87 ÷ 29 = 3 half-lives → 100 × 1/8 = 12.5 g)."*

Final takeaway table (show on screen / read aloud):

You learned...	Household Example
Alpha decay → $^{4}_{2}He$ emitted	Americium-241 in smoke detector
Beta decay → $^{0}_{-1}e$ emitted	Carbon-14 dating
Gamma decay → γ photon emitted	Medical X-rays, CT scans
Half-life: $N = N_0 \times (1/2)^n$	Carbon-14: 5,730 years
Decay constant: $\lambda = 0.693 / t_{1/2}$	Used in $N = N_0e^{-tt}$
Binding energy = energy holding nucleus together	Iron-56 is most stable; explains fission/fusion
Fission = splitting heavy nucleus	Nuclear power plants
Fusion = combining light nuclei	The sun (hydrogen → helium)
Radon test ($15-30 at hardware store)	#1 cause of lung cancer in non-smokers
Background radiation (~2-3 mSv/year)	Safe; from soil, radon, cosmic rays

Final line (preview of L2-12): "Next session: Spectroscopy — how scientists identify chemicals using light, why UV light kills germs, and how a breathalyzer detects alcohol. See you then."

SUPPLEMENTARY MATERIALS FOR L2-11 (No Grade)

Resource	Household Connection	Description	How to Find It
PhET "Alpha Decay" simulation	Nuclear decay visual	Watch alpha particles emitted	Search "PhET alpha decay"
PhET "Beta Decay" simulation	Nuclear decay visual	Watch beta particles emitted	Search "PhET beta decay"
Radon test kit information	Home safety	EPA guidelines	Search "EPA radon test kit"
Half-life calculator	Practice problems	Online calculator	Search "half-life calculator"

"This week, look at the smoke detector in your home. Find the radiation symbol or the word 'americium' on the label (usually on the back). Don't open it — just look. Next

time, tell us what you found."